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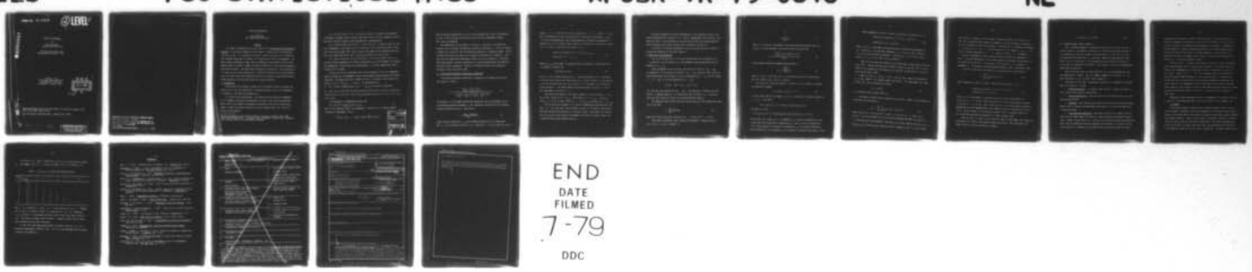

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TESTS FOR DEPENDENCE

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TESTS FOR DEPENDENCE[1]

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Abstract

This paper is prepared as an invited entry for the Encyclopedia of Statistical Sciences, to be edited by Samuel Kotz and Norman L. Johnson and to be published by John Wiley & Sons. It is designed to provide a sound introduction for a reasonably well-informed reader who is, however, not a specialist in tests for dependence. The paper contains references to many tests but emphasizes the parametric test of independence based on Pearson's sample correlation coefficient r and certain nonparametric tests based on ranks. The ranks tests are generally preferable to the test based on r in that they have wider applicability, are much less sensitive to outlying observations, are exact under mild assumptions which do not require an underlying bivariate normal population, and have good efficiency (power) properties.

1. Introduction

Many studies are designed to explore the relationship between two random variables X and Y, say, and specifically to determine whether X and Y are independent or dependent. Some particular examples are:

(i) *Obesity and blood pressure:* Are obesity and blood pressure independent or, for example, do men who are overweight also tend to have high blood pressure? Here X could be the degree of overweight as measured by the ratio of actual body weight to ideal body weight as given in certain standard tables, and Y could be systolic blood pressure.

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(ii) *Color and taste of tuna:* Are color and quality of canned tuna independent or perhaps do consumers tend to prefer light tuna? Here X could be a measure of lightness and Y could be a quality score determined by a consumer panel.

(iii) *Infants walking and their IQ:* Is the time until it takes an infant to walk alone independent of the infant's IQ at a later age, or do children who learn to walk early tend to have higher IQs? Here X could be the number of days measured from birth until the infant walks alone, and Y could be the infant's IQ score at age 5.

(iv) *System reliability and the environment:* Is the life length X (say) of a specific system independent of a certain characteristic of the environment, for example, the temperature Y, within which the system operates, or do high temperatures tend to shorten the life length?

One can test the null hypothesis that the two variables X and Y are independent, against alternatives of dependence, using a random sample from the underlying bivariate population. We suppose that such a sample of size n is available, and we denote the sample by (X_1,Y_1), (X_2,Y_2), ..., (X_n,Y_n). Our assumptions are

A1. *The n bivariate observations* $(X_1,Y_1),\ldots,(X_n,Y_n)$ *are mutually independent.*

A2. *Each* (X_i,Y_i) *comes from the same bivariate population with continuous distribution function* $H(x,y) = P(X\le x, Y\le y)$ *and continuous marginal distributions* $F(x) = P(X\le x)$ *and* $G(y) = P(Y\le y)$.

The *hypothesis of independence* asserts that

$$H_0\colon\ H(x,y) = F(x)G(y), \text{ for all } (x,y), \tag{1}$$

that is, the variables X and Y are independent. Under H_0, all 2n random variables are mutually independent, that is

$$P(X_1\le x_1,\ Y_1\le y_1,\ \ldots,\ X_n\le x_n,\ Y_n\le y_n) = \prod_{i=1}^{n} F(x_i)G(y_i).$$

When we discuss alternatives to H_0, we will be assuming that X and Y are dependent so that (1) fails to hold, but we still insist that the independence *between* the n pairs is preserved.

The organization of this paper is as follows. In Section 2 we present the classical test of H_0 based on Pearson's correlation coefficient r. This test assumes, in addition to A1 and A2, that the underlying population is bivariate normal. Section 3 present rank tests of H_0 which do not require the assumption of normality. These rank tests have additional advantages, relative to the test based on r, including wider applicability, relative insensitivity to outlying observations, and desirable efficiency (power) properties. Section 4 illustrates various tests using data relating to color and taste of tuna.

2. A Test Based on Pearson's Correlation Coefficient

The Pearson correlation coefficient r, proposed by the eminent statistician Karl Pearson in 1896, is

$$r = \frac{n \sum_{i=1}^{n} X_i Y_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\{[n \sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2][n \sum_{i=1}^{n} Y_i^2 - (\sum_{i=1}^{n} Y_i)^2]\}^{\frac{1}{2}}} . \tag{2}$$

The statistic r is the sample correlation coefficient and is an estimator of the corresponding population parameter ρ, the correlation coefficient of the bivariate population defined by H(x,y). Specifically,

$$\rho = \frac{E(XY) - E(X)E(Y)}{\sigma_x \sigma_y} , \tag{3}$$

where E denotes expectation, σ_x is the standard deviation of the X population, and σ_y is the standard deviation of the Y population. It can be shown that for all samples $-1 \leq r \leq 1$, and for all bivariate populations $-1 \leq \rho \leq 1$. When $\rho > 0$, this may be interpreted as X and Y being positively associated (as measured by ρ) and $\rho < 0$ may be interpreted as X and Y being negatively associated (as measured by ρ). *Assuming* H(x,y) *is a bivariate normal cumulative distribution function with correlation* ρ, an exact α level test of H_0 versus $\rho \neq 0$ is

$$\begin{aligned}&\text{reject } H_0 \text{ in favor of } \rho \neq 0 \text{ if } |T| \geq t_{\alpha/2,n-2},\\ &\text{accept } H_0 \text{ if } |T| < t_{\alpha/2,n-2},\end{aligned} \tag{4}$$

where $t_{\alpha/2,n-2}$ is the upper $\alpha/2$ percentile point of Student's t distribution with n-2 degrees of freedom, and

$$T=(n-2)^{\frac{1}{2}}r/(1-r^2)^{\frac{1}{2}}. \tag{5}$$

Since $|T|$ is an increasing function of $|r|$, the test defined by (4) is equivalent to the test which rejects for large values of $|r|$, and the latter is easily derived to be the likelihood ratio test of H_0 versus $\rho \neq 0$ in the model which assumes bivariate normality. (Of course under the bivariate normality assumption, X and Y are independent if and only if $\rho = 0$.)

One-sided tests based on T are readily defined. To test H_0 versus $\rho > 0$, at the α level, reject H_0 if $T \geq t_{\alpha,n-2}$ and accept H_0 if $T < t_{\alpha,n-2}$. To test H_0 versus $\rho < 0$, at the α level, reject H_0 if $T \leq -t_{\alpha,n-2}$ and accept H_0 if $T > -t_{\alpha,n-2}$.

The two-sided test defined by (5), and the corresponding one-sided tests, are exact (i.e., have true Type I error probability equal to the nominal value value α) only when the underlying population is bivariate normal. Approximate (for large n) tests of H_0 which do not require the assumption of bivariate normality treat T as a standard normal random variable under H_0.

For more information on testing independence in this parametric context, see Bickel and Doksum (1977, Section 6.5.A). Devlin, Gnanadesikan, and Kettenring (1975) point out that r is very sensitive to outliers and consider the related problem of robust estimation and outlier detection with correlation coefficients.

In Section 3 we present nonparametric tests of H_0 which are exact without requiring the assumption of bivariate normality.

3. Rank Tests of Independence

Let R_i be the rank of X_i in the joint ranking from least to greatest of $X_1, \ldots, X_n$ and let S_i be the rank of Y_i in the (separate) joint ranking from least to greatest of $Y_1, \ldots, Y_n$.

Under assumptions A1 and A2 and H_0, the vector of X ranks $R = (R_1, \ldots, R_n)$ is independent of the vector of Y ranks $S = (S_1, \ldots, S_n)$, and both R and S have uniform distributions over the space $\mathcal{P}$ *of the* $n!$ permutations $(i_1, \ldots, i_n)$ *of the integers* $(1, \ldots, n)$. That is, for each permutation $(i_1, \ldots, i_n)$,

$$P_0\{(R_1, \ldots, R_n) = (i_1, \ldots, i_n)\} = 1/n!,$$

with the same result holding for $(S_1, \ldots, S_n)$. (The subscript 0 indicates the probability is computed under H_0.) It follows that rank statistics (i.e., statistics which are solely based on R and S) are distribution-free under H_0.

One important class of rank statistics for testing H_0 are the *linear* rank statistics of the form

$$L = \sum_{i=1}^{n} a(R_i) b(S_i) \tag{6}$$

where the "scores" $a(R_i), b(S_i)$ satisfy $a(1) \le \ldots \le a(n)$, $b(1) \le \ldots \le b(n)$.

Test based on Spearman's rank correlation coefficient: Making the choice $a(i) = b(i) = i$ in (6), L reduces to

$$M = \sum_{i=1}^{n} R_i S_i . \qquad (7)$$

Then if M is linearily transformed so that the minimum and maximum values are -1 and 1, we obtain Spearman's rank order correlation coefficient

$$r_s = \frac{12 \sum_{i=1}^{n} [R_i - (n+1)/2][S_i - (n+1)/2]}{n(n^2-1)} . \qquad (8)$$

An even simpler formula for computational purposes is

$$r_s = 1 - \frac{6 \sum_{i=1}^{n} D_i^2}{n^3 - n} , \qquad (9)$$

where $D_i = R_i - S_i$. Note also that r_s is obtainable from r(2) by replacing X_i with its X-rank R_i and Y_i with its Y-rank S_i.

The statistic r_s does not estimate ρ as given in (3) but rather it estimates the population parameter

$$\rho_s = 6P\{(X_1-X_2)(Y_1-Y_3) > 0\} - 3. \qquad (10)$$

It can be shown that for all samples $-1 \le r_s \le 1$, and for all bivariate populations $-1 \le \rho_s \le 1$. Note that

$$P\{(X_1-X_2)(Y_1-Y_3) > 0\} = P(X_1>X_2, Y_1>Y_3) + P(X_1<X_2, Y_1<Y_3)$$

and when H_0 is true

$$P\{(X_1-X_2)(Y_1-Y_3) > 0\} = P(X_1>X_2)P(Y_1>Y_3) + P(X_1<X_2)P(Y_1<Y_3) = \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2},$$

so that when H_0 is true, $\rho_s = 0$. In addition, $\rho_s > 0$ may be interpreted as X and Y being positively associated (as measured by ρ_s), and $\rho_s < 0$ may be interpreted as X and Y being negatively associated (as measured by ρ_s). (For further information and interpretation of the parameter ρ_s as a measure of association see Kruskal (1958).)

Under assumptions A1 and A2, an exact α level test of H_0 versus $\rho_s \neq 0$ is

$$\text{reject } H_0 \text{ in favor of } \rho_s \neq 0 \text{ if } |r_s| \geq r_s(\alpha/2,n), \qquad (11)$$
$$\text{accept } H_0 \text{ if } |r_s| < r_s(\alpha/2,n),$$

where $r_s(\alpha/2,n)$ is the upper $\alpha/2$ percentile point of the null distribution of r_s. To test H_0 vs. the one-sided alternative $\rho_s > 0$, at the α level, reject H_0 if $r_s \geq r(\alpha,n)$ and accept H_0 otherwise. To test H_0 vs. $\rho < 0$, at the α level, reject H_0 if $r_s \leq -r(\alpha,n)$ and accept H_0 otherwise.

From (9) we see that tests based on r_s are equivalent to tests based on the statistic ΣD_i^2. Glasser and Winter give critical values of r_s and ΣD_i^2 for n = 4(1)30. Tables of the complete null distribution of r_s and ΣD_i^2 are given for n = 4(1)11 in Kraft and van Eeden (1968).

Under H_0, $E(r_s) = 0$, $Var(r_s) = 1/(n-1)$, and as n gets large, the distribution of $(n-1)^{\frac{1}{2}} r_s$ tends to the standard normal distribution. Thus approximate (for large n) tests of H_0 can be obtained by treating

$$r_s^* = (n-1)^{\frac{1}{2}} r_s \qquad (12)$$

as a standard normal variable under H_0.

Test based on Kendall's rank correlation coefficient: Kendall's rank correlation coefficient can be written as

$$r_k = 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \xi(X_i,X_j,Y_i,Y_j)/\{n(n-1)\}, \qquad (13)$$

where $\xi(a,b,c,d) = 1$ if $(a-b)(c-d) > 0$, and $= -1$ if $(a-b)(c-d) < 0$. When $(X_i-X_j)\cdot(Y_i-Y_j) > 0$ we say the pairs $(X_i,Y_i),(X_j,Y_j)$ are concordant and when $(X_i-X_j)(Y_i-Y_j) < 0$ we say the pairs are discordant. Note that r_k is a rank statistic ($\xi(X_i,X_j,Y_i,Y_j) = \xi(R_i,R_j,S_i,S_j)$ so that one only needs the ranks to compute r_k) but it is not a linear rank statistic. However, it can be shown (cf. Hájek and Šidák, 1967, Section II.3.1) that, up to a multiplicative constant, Spearman's r_s is the "projection" of Kendall's r_k into the family of linear rank statistics. The statistic r_k estimates the parameter $\tau = 2P\{(X_1-X_2)(Y_1-Y_2) > 0\}-1$. It can be shown that for all samples $-1 \le r_k \le 1$, and for all bivariate populations $-1 \le \tau \le 1$. When H_0 is true, $\tau = 0$. In addition, $\tau > 0$ may be interpreted as X and Y being positively associated (as measured by τ), and $\tau < 0$ may be interpreted as X and Y being negatively associated (as measured by τ). The reader should note that τ is analogous to the parameter ρ_s(10) estimated by Spearman's r_s. For details of the relationship between ρ_s and τ, see Kruskal (1958).

From (13) we see that tests based on r_k are equivalent to tests based on

$$K = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \xi(X_i, X_j, Y_i, Y_j). \tag{14}$$

Under assumptions A1 and A2, an exact α level test of H_0 versus $\tau \ne 0$ is

$$\begin{aligned} &\text{reject } H_0 \text{ in favor of } \tau \ne 0 \text{ if } |K| \ge k(\alpha/2,n), \\ &\text{accept } H_0 \text{ if } |K| < k(\alpha/2,n), \end{aligned} \tag{15}$$

where $k(\alpha/2,n)$ is the upper $\alpha/2$ percentile point of the null distribution of K. To test H_0 vs. $\tau > 0$, at the α level, reject H_0 if $K \ge k(\alpha,n)$ and accept H_0 otherwise. To test H_0 vs. $\tau < 0$, at the α level, reject H_0 if $K \le -k(\alpha,n)$ and accept H_0 otherwise. Kaarsemaker and van Wijngaarden (1953) give tables of the null distribution of K for n= 4(1)40. See also Table A.21 of Hollander and Wolfe (1973). Extended tables up to n = 100 are made available on request by D.J. Best, see Best (1973).

Under H_0, $E(K) = 0$, $Var(K) = n(n-1)(2n+5)/18$, and as n gets large, the standardized distribution of K tends to the standard normal distribution. Thus approximate (for large n) tests of H_0 can be obtained by treating

$$K^* = K/[n(n-1)(2n+5)/18]^{\frac{1}{2}} \tag{16}$$

as a standard normal variable under H_0.

Ties: Although assumption A2 precludes the possibility of ties, ties may occur in practice. One method of treating ties, when dealing with rank statistics, is to replace R_i by R_i^* (the average of the ranks that X_i is tied for), S_i by S_i^* (the average of the ranks that Y_i is tied for), compute the rank statistic using the R^*'s and S^*'s, and refer it to the appropriate null distribution tables derived under the assumption of continuity. This, however, yields only an approximate, rather than an exact test.

Exact conditional tests, in the presence of ties, can be performed but they are computationally tedious. See, for example, Lehmann (1975, Section 7.3). For more information on ties, see Hájek (1969, Chapter VII).

Advantages of rank tests: Advantages of rank tests, as compared to the parametric test based on r, include:

1. Wider applicability - To compute a rank statistic, we need only know the ranks, rather than the actual observations.

2. Outlier insensitivity - Rank statistics are less sensitive than r to wildly outlying observations.

3. Exactness - Tests based on rank statistics are exact under the mild assumptions A1 and A2, whereas the significance test based on r is exact only when $H(x,y)$ is bivariate normal.

4. Good efficiency properties - Rank tests of H_0 are only slightly less efficient than the normal theory test based on r under the underlying bivariate population is normal (the home court of r), and they can be mildly and wildly more efficient than r when the underlying bivariate population is not normal. Of course, the efficiency question is complicated as it depends both on the specific rank test under

consideration and the specific measure of efficiency used. Roughly speaking, for large n and dependency alternatives "close" to the null hypothesis, the tests based on r_s and r_k sacrifice nine percent of the information in the sample, as compared to the test based on r, under the underlying population in bivariate normal, and can be much more efficient for certain non-normal populations. For more details on efficiency and power, see Lehmann (1975), Section 7.5E) and Hájek and Šidák (1967, Section VII. 2.4), and the references therein.

Other rank tests: A "normal scores" rank test studied by Fieller and Pearson (1961) and Bhuchongkul (1964) is particularly noteworthy. The normal scores test statistic for independence is a linear rank statistic of the form (6) with $a(i) = b(i) = EV_n^{(i)}$ where $V_n^{(1)} < \ldots < V_n^{(n)}$ is an ordered sample of n observations from the standard normal distribution. For a suitable choice of the definition of efficiency and a suitable choice of the nature of dependency alternatives, the normal scores test of independence and the test based on r are equally efficient under "normality" and Srivastava (1973) has shown that the normal scores test is more efficient than the test based on r for "all" (i.e. subject to mild regularity) other cases.

References to other nonparametric tests of independence can be found in Sections 8.1 and 10.2 of Hollander and Wolfe (1973) and in Section 7.5D of Lehmann (1975).

3. Example

The following example is based on data of Rasekh, Kramer, and Finch (1970) in a study designed to ascertain the relative importance of the various factors contributing to tuna quality and to find objective methods for determining quality parameters and consumer preference. Table 1 gives values of the Hunter L measure of lightness, along with panel scores for nine lots of canned tuna. The original consumer panel scores of excellent, very good, good, fair, poor, and unacceptable were converted to the numerical values of 6,5,4,3,2, and 1, respectively. The panel scores in Table 1 are averages of 80 such values. The Y random variable is thus discrete, and hence the continuity portion of assumption A2 is not satisfied. Nevertheless, since each Y is an average of 80 values, we need not be too nervous about this departure from assumption A2.

It is suspected that the Hunter L value is positively associated with the panel score. Thus we will illustrate the one-sided tests of H_0 versus positive association, based on r, r_s, and r_k. The reader will soon see that all three tests reach the same conclusion, i.e., there is positive association between the Hunter L value and the panel score.

Table 1. Hunter L values and consumer panel scores for nine lots of canned tuna.

Lot	Hunter L Value (X)	Panel Score (Y)
1	44.4	2.6
2	45.9	3.1
3	41.9	2.5
4	53.3	5.0
5	44.7	3.6
6	44.1	4.0
7	50.7	5.2
8	45.2	2.8
9	60.1	3.8

Source. J. Rasekh, A. Kramer, and R. Finch (1970).

Test based on r: From Table 1, we easily calculate $\Sigma X_i Y_i = 1584.88$, $\Sigma X_i = 430.3$, $\Sigma Y_i = 32.6$, $(\Sigma X_i)^2 = 185158.09$, $(\Sigma Y_i)^2 = 1062.76$, $\Sigma X_i^2 = 20843.11$, $\Sigma Y_i^2 = 125.90$, and from (2) and (5) with n = 9, we obtain r = .57 and T = 1.84. Referring T = 1.84 to a t-distribution with 7 degrees of freedom yields a one-sided P value of .054. Thus the test based on r leads to the conclusion that the Hunter L lightness variable and the panel score variable are positively associated.

The large sample approximation refers T = 1.84 to the standard normal distribution yielding an approximate P value of .034.

Test based on r_s: We use Table 2 to illustrate the computation of r_s.

Table 2. Computation of r_s for the canned tuna data

Lot	R	S	D	D^2
1	3	2	1	1
2	6	4	2	4
3	1	1	0	0
4	8	8	0	0
5	4	5	-1	1
6	2	7	-5	25
7	7	9	-2	4
8	5	3	2	4
9	9	6	3	9
				$\Sigma D^2=48$

From (9) with n = 9 we obtain

$$r_s = 1 - \frac{6(48)}{(9)^3-9} = .60.$$

Referring r_s = .60 to Table J of Kraft and van Eeden (1968) yields a one-sided P value of .048. Thus the test based on r_s leads to the conclusion that the Hunter L lightness variable and the panel score variable are positively associated.

From (12) we see that the large sample approximation refers $r_s^* = (8)^{\frac{1}{2}}(.6) =$ 1.70 to the standard normal distribution yielding an approximate P value of .045. This is in good agreement with the exact P value of .048 based on r_s.

Test based on r_k: Table 3 contains the $\xi(X_i, X_j, Y_i, Y_j)$ values used to compute r_k. For example, the i = 2, j = 5 entry in Table 3 is a "-1" because $X_2 > X_5$

Table 3. $\xi(X_i, X_j, Y_i, Y_j)$ values for canned tuna data

j \ i	1	2	3	4	5	6	7	8
2	1							
3	1	1						
4	1	1	1					
5	1	-1	1	1				
6	-1	-1	1	1	-1			
7	1	1	1	-1	1	1		
8	1	1	1	1	-1	-1	1	
9	1	1	1	-1	1	-1	-1	1

and $Y_2 < Y_5$, yielding $(X_2 - X_5)(Y_2 - Y_5) < 0$ and thus $\xi(X_2, X_5, Y_2, Y_5) = -1$. Summing the 1's and -1's of Table 3 yields K = 16 and from (13), $r_k = .44$. Referring K = 16 to Table A.21 of Hollander and Wolfe (1973) yields a one-sided P value of .060. Thus there is evidence that the Hunter L lightness variable and the panel score variable are positively associated.

To apply the large sample approximation we compute, from (16), $K^* = 1.67$ yielding an approximate P value of .048. This is in good agreement with the exact P value of .060 based on K.

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This paper is prepared as an invited entry for the Encyclopedia of Statistical Sciences, to be edited by Samuel Kotz and Norman L. Johnson and to be published by John Wiley & Sons. It is designed to provide a sound introduction for a reasonably well-informed reader who is, however, not a specialist in tests for dependence. The paper contains references to many tests but emphasizes the parametric test of independence based on Pearson's sample correlation coefficient r and certain nonparametric tests based on ranks. The ranks tests are generally preferable to the test based on r in that they have wider applicability, are much less sensitive to outlying observations, are exact under mild assumptions which do not require an underlying bivariate normal population, and have good efficiency (power) properties.

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This paper is prepared as an invited entry for the Encyclopedia of Statistical Sciences, to be edited by Samuel Kotz and Norman L. Johnson and to be published by John Wiley & Sons. It is designed to provide a sound introduction for a reasonably well-informed reader who is, however, not a specialist in tests for dependence. The paper contains references to many tests but emphasizes the parametric test of independence based on Pearson's sample correlation coefficient r and certain nonparametric tests based on ranks. The ranks tests are generally preferable to the test based on r in

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20. Abstract continued.

that they have wider applicability, are much less sensitive to outlying observations, are exact under mild assumptions which do not require an underlying bivariate normal population, and have good efficiency (power) properties.